STRONG INTERACTION BETWEEN THE BOUNDARY LAYER AND THE INVISCID FLOW PAST A TRIANGULAR WING (*)

(O SIL'NOM VZAIMODEISTVII POGRANICHNOGO SLOIA S NEVIAZKIM POTOKOM NA TREUGOL'NOM KRYLE)

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M.D.LADYZHENSKII

(Moscow)

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The paper considers the viscous hypersonic flow past an infinitely slender triangular wing at zero angle of attack and free-stream Mach number $M_\infty = \infty$. It has been shown previously in [1], that the solution to the equations of the three-dimensional boundary layer, obtained independently of the left and right wing edges, is identical with the solution for strong interaction on a flat plate with slip flow. In view of the fact that the system of equations is parabolic this solution does not satisfy the condition of symmetry of flow in the plane of symmetry of the wing and does not hold in that region. In the present paper we shall construct a solution in the neighborhood of the plane of symmetry of the wing. Due to the fact that the secondary flows in the boundary layer are directed towards the plane of symmetry of the wing, the thickness of the effective body determined by the displacement of the boundary layer increases. The thickening of the effective body results in a strengthening of the shock and in an increase of pressure as compared with the value obtained from the solution for slip flow over a plate. It is shown that when the Reynolds number tends to infinity the transverse cross-section of the effective body in a plane normal to the undisturbed flow tends to a semicircle.

1. Consider the equations of a three-dimensional boundary layer in a Cartesian system of coordinates xyz (the x-axis passes through the apex of the triangular wing and is parallel to the undisturbed velocity vector U_∞ (Fig.1), the y-axis is normal to the plane of the wing).

Let us introduce the following notation: u, v, w — components of the velocity vector in the x, y, z directions, respectively, p — pressure, ρ — density, i — enthalpy, i_0 — stagnation enthalpy, μ — coefficient of

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*) The present issue was in press when the editors received the sad news about the tragic untimely death of the gifted young scientist. A biographical sketch and a list of publications of the author are given at the end of this paper.

Editors.

viscosity, $\varkappa$ — adiabatic exponent (the gas is assumed to be perfect), σ — Prandtl number. In the case of flow over a triangular plate at zero angle of attack, the system of equations of the three-dimensional boundary layer has a self-similar solution [1] which is a function of the two variables η and ζ only

$$u = U_\infty U(\eta, \zeta), \qquad v = R_x^{-1/4} U_\infty V(\eta, \zeta), \quad w = U_\infty W(\eta, \zeta)$$
$$p = R_x^{-1/2} \rho_\infty U_\infty^2 P(\zeta), \quad \rho = R_x^{-1/2} \rho_\infty R(\eta, \zeta), \quad i = U_\infty^2 h(\eta, \zeta) \qquad (1.1)$$
$$i_0 = U_\infty^2 H(\eta, \zeta), \qquad \eta = R_x^{1/4} y/x, \ \zeta = z/x, \quad R_x = \rho_\infty U_\infty x/\mu_0$$

Here ρ_∞ is the density of the undisturbed flow and μ_0 is the coefficient of viscosity corresponding to the stagnation temperature. The equation of the outer edge of the boundary layer is

$$y = \delta(x, z) = x R_x^{-1/4} \Phi(\zeta) \qquad (1.2)$$

The functional relation between the dimensionless pressure $P(\zeta)$ and the function $\Phi(\zeta)$ is taken from the solution to the equations of inviscid flow separated from the viscous flow by a sharp boundary, which can be obtained by the "strip theory" [2]. After substitution of (1.1) the equations of the three-dimensional boundary layer reduce to a system of equations of parabolic type, for which ζ is a characteristic.

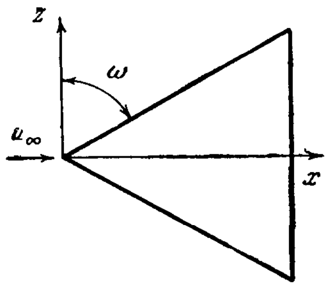

Fig. 1

Due to the fact that the equations are parabolic, the solution (which is constructed starting from the leading edge) is identical with the solution for strong interaction on a flat plate with slip. This solution, which is a function of one variable only, is [1]

$$U = \chi'(\lambda), \quad W = \psi'(\lambda) + \zeta_0 \chi'(\lambda), \qquad H = g(\lambda)$$
$$\psi\psi'' + 2\varepsilon(1 + \zeta_0^2) L = 4\varepsilon^{-1}\psi''' \qquad (\varepsilon = (\varkappa - 1)/2\varkappa)$$
$$\psi\chi'' - 2\varepsilon\zeta_0 L = 4\varepsilon^{-1}\chi''' \qquad (\zeta_0 = \cot\omega) \qquad (1.3)$$
$$g'\psi = \frac{4}{\varepsilon}\left(\frac{g'}{\sigma}\right)' + \frac{4}{\varepsilon}\frac{d}{d\lambda}\left[\left(1 - \frac{1}{\sigma}\right)\frac{d}{d\lambda}\frac{(2g - L)}{2}\right]$$
$$L = 2g - (1 + \zeta_0^2)(\chi')^2 - (\psi')^2 - 2\zeta_0\psi'\chi'$$
$$\psi(0) = \psi'(0) = \chi(0) = \chi'(0) = 0, \quad \psi'(\infty) = -\zeta_0, \ \chi'(\infty) = 1$$
$$g(0) = g_b \quad (\text{or } g'(0) = 0), \qquad g(\infty) = 1/2$$

Here ω is the angle of sweep of the leading edge of the wing, and g_b is the enthalpy which corresponds to the temperature of the wall. The variable λ is connected with η and ζ by A.A.Dorodnitsyn's transformation

$$\lambda = \frac{1}{\sqrt{A}(\zeta_0 - \zeta)^{1/4}} \int_0^\eta R d\eta, \quad \text{or} \quad \frac{\eta}{(\zeta_0 - \zeta)^{3/4}} = \frac{\varepsilon}{\sqrt{A}} \int_0^\lambda L(\lambda)\, d\lambda \qquad (1.4)$$

$$A = \frac{3\varepsilon\zeta_0}{4} \sqrt{C} \int_0^\infty L(\lambda)\, d\lambda$$

where C is the constant in Equation $p = \rho_\infty U_\infty^2 C\theta^2$, which relates the pressure in the inviscid flow with the angle of inclination θ between the outer edge of the boundary layer and the x-axis. The inviscid flow generated at the leading edge corresponds to flow over a power-law body $y = a\,(x \cos\omega - z \sin\omega)^{3/4}$, where a is a constant. Thus, instead of the tangent-wedge approximation $C = \frac{1}{2}(\varkappa + 1)$ [1] we can take the exact value of C obtained from the solution of the self-similar inviscid flow, $C = 1.42$ for $\varkappa = 7/5$ and $C = 1.77$ for $\varkappa = 5/3$ (cf., e.g., [2], p.455 of the Russian translation).

For simplicity we assume in Equations (1.3) a linear dependence of viscosity on enthalpy $\mu/\mu_0 = 2h$. In the case of flow over an insulated plate ($G'(0) = 0$) and $\sigma = 1$ Equations (1.3) reduce to a simpler form due to the existence of the integral $G = \frac{1}{2}$. The dimensionless pressure P, density R, and boundary-layer thickness Φ are determined by the relations

$$P = \frac{A}{(\zeta_0 - \zeta)^{1/2}}, \quad R = \frac{P}{\varepsilon L}, \quad \Phi = B(\zeta_0 - \zeta)^{3/4}, \quad B = \frac{4\sqrt{A}}{3\zeta_0 \sqrt{C}} \qquad (1.5)$$

(R is found from the equation of state). The solution for (1.3) to (1.5) does not satisfy the condition $w = 0$ at $z = 0$, i.e. it is not valid near the plane of symmetry of the wing. In the boundary layer the z-component of the velocity vector is directed towards the plane of symmetry of the wing [1].

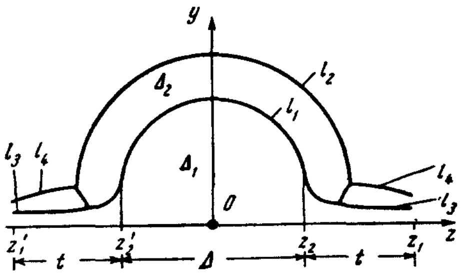

Fig. 2

As a result of the collision of the flows from the left and right leading edges, the effective body formed by the outer edge of the boundary layer becomes thicker. Let Δ denote a characteristic transverse dimension of the region near the plane of symmetry inside which the solution (1.3), (1.4) and (1.5) does not hold, and let us call that region the Δ-region.

Let the Δ_1-region be that part of the Δ-region which is filled with gas coming from the boundary layer, and let the Δ_2-region be the region of inviscid flow over the effective body Δ_1. Fig.2 represents the assumed flow regions in the plane $x = \text{const}$. Here l_1 is the boundary separating Δ_1 and Δ_2, l_2 is the shock wave which separates the Δ_2-region from the undisturbed flow, and l_3, l_4 are the outer edge of the boundary layer and the shock wave corresponding to slip flow over a flat plate, respectively.

The flow parameters in the Δ_1-region can be estimated as

$$y \sim \Delta, \quad z \sim \Delta, \quad u \sim U_\infty, \quad v \sim \frac{U_\infty \Delta}{x}$$
$$w \sim \frac{U_\infty \Delta}{x}, \qquad p \sim \frac{\rho_\infty U_\infty^2 \Delta^2}{x^2}, \qquad i \sim U_\infty^2, \ \rho \sim \frac{\rho_\infty \Delta^2}{x^2} \tag{1.6}$$

The estimates for u and i, which are the usual boundary-layer estimates, are subsequently confirmed by the integral-method solution for the flow in the Δ-region. The estimates for v and w follow from the continuity equation. The pressure p (as well as the flow parameters in the Δ_2-region) is estimated by the usual hypersonic-flow method as the pressure corresponding to flow over a slender body of thickness Δ, and the estimate for ρ is obtained from the equation of state (ϵ is not assumed to be small).

Let us estimate the total mass flow from the boundary layer into the Δ_1-region

$$Q = \int_0^x dx \int_0^\delta \rho(x, y, 0)\, W(x, y, 0)\, dy, \quad \delta = \delta(x, 0) \tag{1.7}$$

Here $\delta = \delta(x,z)$ is the boundary-layer thickness. As will be seen later, $\Delta/x \to 0$ for $R_x \to \infty$. Therefore the mass flow Q is determined from the solution for slip flow over a flat plate, evaluated at $z = 0$. This yields the estimates (δ_* is the characteristic boundary-layer thickness)

$$\rho \sim \rho_\infty \delta_*^2/x^2, \quad w \sim U_\infty, \quad Q \sim \rho_\infty U_\infty \delta_*^3/x \quad (\delta_*/x = R_x^{-1/4}) \tag{1.8}$$

Now we equate Q with Q' – the mass flow through the Δ_1-region in the x-direction, using estimates (1.6) and (1.8)

$$Q \doteq Q', \qquad \rho_\infty U_\infty \delta_*^3/x \sim \rho_\infty U_\infty \Delta^4/x^2, \qquad \Delta/x \sim (\delta_*/x)^{3/4} \tag{1.9}$$

Thus, when $R_x \to \infty$ the ratio Δ/x tends to zero more slowly than δ_*/x, so that the ratio $\Delta/\delta_* \to \infty$. In the following we investigate the assymptotic behavior of the solution for $\Delta/\delta_* \gg 1$.

Let us estimate the ratio between the convective and viscous terms in the Δ_1-region. Using (1.6) and (1.9) in the x-momentum equation, we obtain the estimates for the sum of the convective terms K and for the largest viscous term V_μ

$$K \sim \rho u \frac{\partial u}{\partial x} \sim \frac{\rho_\infty U_\infty^2}{x}\left(\frac{\Delta}{x}\right)^2, \qquad V_\mu = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \sim \frac{\mu_0 U_\infty}{\Delta^2}$$
$$\frac{V_\mu}{K} = \frac{\mu_0 U_\infty}{\Delta^2} \Big/ \frac{\rho_\infty U_\infty^2}{x}\left(\frac{\Delta}{x}\right)^2 = \frac{\mu_0}{\rho_\infty U_\infty x}\left(\frac{x}{\Delta}\right)^4 \sim \frac{\delta_*}{x} \tag{1.10}$$

Analogous ratios between the convective and the dissipative terms in the Δ_1-region hold for the other components of the momentum equation and for the energy equation. The essential result is that the flow in the Δ_1-region is inviscid. Clearly, inside the Δ_1-region the no-slip condition must be satisfied at the wall and a boundary layer forms near the wall. It is important to note, however, that the thickness of this boundary layer is much less than Δ.

Substituting (1.6), (1.9) in the y- and z-momentum equations we obtain the estimates for the characteristic pressure drops Δp_y and Δp_z across the Δ_1-region

$$\Delta p_y = \frac{\partial p}{\partial y}\Delta \sim \rho u \frac{\partial v}{\partial x}\Delta \sim \rho_\infty U_\infty^2 \left(\frac{\Delta}{x}\right)^4, \quad \Delta p_z = \frac{\partial p}{\partial z}\Delta \sim \rho u \frac{\partial w}{\partial x}\Delta \sim \rho_\infty U_\infty^2 \left(\frac{\Delta}{x}\right)^4$$

$$\frac{\Delta p_y}{p} \sim \left(\frac{\Delta}{x}\right)^2 \sim \left(\frac{\delta_*}{x}\right)^{3/2}, \qquad \frac{\Delta p_z}{p} \sim \left(\frac{\Delta}{x}\right)^2 \sim \left(\frac{\delta_*}{x}\right)^{3/2} \tag{1.11}$$

The relative change of pressure across the Δ_1-region tends to zero as R_x tends to infinity, i.e. within a relative error of order $(\delta_* / x)^{3/2}$ the pressure in the Δ_1-region depends only on x. Hence it follows that the boundary l_1 between the regions Δ_1 and Δ_2 in the plane x = const (Fig.2) is a semicircle. This is the only case in which the pressure in the Δ_2-region is constant along l_1, like in the case of flow past an axisymmetric body whose axis coincides with the x-axis. The axisymmetry of the flow in the Δ_2-region is established in a manner analogous to that used in [3 and 4] in the analysis of the flow in the entropy and boundary layers over elongated slender bodies.

2. The thickening of the effective body near the plane of symmetry leads to the appearance of a shock wave l_2 (Fig.2) which is semicircular in the x = const plane. We obtain a system of equations which relate the flow parameters at the beginning (section z_1 in Fig.2) and at the end (section z_2 in Fig.2) of the region of interaction of the shock wave l_2 with the boundary layer. In this region, whose width t tends to zero as R_x tends to infinity, the boundary-layer equations do not hold and one must use the Navier-Stokes equations.

Inside this t-region (as we shall call it) the flow changes form the boundary-layer slip flow over a flat plate to the Δ-region flow. The characteristic values of the flow parameters in the t-region are intermediate between those in the boundary layer and in the Δ-region. The density in the t-region is low, as in the boundary layer $\rho \sim \rho_\infty \delta^2_* / x^2$, and in the Δ-region $\rho \sim \rho_\infty (\delta_* / x)^{3/2}$. Therefore the thickness of the region $\delta(x, z)$ (dimension in the y-direction) coincides, as in the case of the boundary layer, with the displacement thickness, i.e. the surface $y = \delta(x, z)$ can be considered to be a stream surface. On this surface the components of the heat-flow vector and the stress tensor are equal to zero. Let us write down the Navie-Stokes equations in divergence form, integrate these over the volume of the t-region (x' is some fixed value)

$$0 \leqslant x \leqslant x', \quad 0 \leqslant y \leqslant \delta(x, z), \quad z_1(x) \leqslant z \leqslant z_2(x) \tag{2.1}$$

and transform the volume integrals into surface integrals.

Taking account of the no-slop condition at the wall and the condition that the viscous stresses and the heat flux vanish on the surface $y = \delta(x,z)$, we obtain

$$\iint \rho v_n dS_1 + \iint \rho v_n dS_2 = 0 \tag{2.2}$$

$$\iint (\rho v_n \mathbf{v} + p\mathbf{n} - T_n)\, dS_1 + \iint (\rho v_n \mathbf{v} + p\mathbf{n} - T_n)\, dS_2 + \\ + \iint p\mathbf{n}\, dS_3 + \iint (p\mathbf{n} - T_n)\, dS_4 = 0 \tag{2.3}$$

$$\iint [\rho v_n i_0 - q_n - (\mathbf{v}T_n)]\, dS_1 + \iint [\rho v_n i_0 - q_n - (\mathbf{v}T)_n]\, dS_2 + \iint q_n dS_4 = 0 \tag{2.4}$$

Here S_1, S_2, S_3, S_4 are the lateral surfaces of the volume under consideration, which are parts of the surfaces $z = z_1(x)$, $z = z_2(x)$, $y = \delta(x, z)$ and $y = 0$, respectively, $\mathbf{n}$ is the outer normal to the lateral surface, v_n is the projection of the velocity vector $\mathbf{v}$ on $\mathbf{n}$, T is the viscous stress tensor, T_n is the stress acting on a unit sutface element with normal $\mathbf{n}$, $(\mathbf{v}T)$ is the product of the vector $\mathbf{v}$ and the tensor T, and q_n, $(\mathbf{v}T)_n$ are the projections of the heat-flow vector $\mathbf{q}$ and of the vector $(\mathbf{v}T)$ on the normal $\mathbf{n}$.

Let δ_*/x tend to zero. In that case the lines $z = z_1(x)$ and $z = z_2(x)$ tend to tne line $z = 0$ as, clearly, $z_1 \sim z_2 \sim \Delta$, and v_n becomes $+w$ on S_1 and to $-w$ on S_2. Equation (2.4) takes the form

$$\iint \rho w\, dS_1 - \iint \rho w\, dS_2 = 0 \tag{2.5}$$

In the following we shall need only the x-momentum equation. On the S_1 surface we have

$$\cos nx = \Delta / x, \quad \cos ny = 0, \quad \cos nz = 1 + O(\Delta^2/x^2) \tag{2.6}$$

$$T_{nx} = \tau_{xx} \cos nx + \tau_{xy} \cos ny + \tau_{xz} \cos nz \sim \tau_{xx}\Delta / x + \tau_{xz}$$

Taking account of the expressions for the viscous stresses we obtain the estimates

$$\tau_{xx} = \frac{4}{3}\mu \frac{\partial u}{\partial x} - \frac{2\mu}{3}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \sim \frac{\mu_0 U_\infty}{t}, \tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \sim \frac{\mu_0 U_\infty}{t} \tag{2.7}$$

Here the operator $\partial/\partial z$ is estimated as t^{-1}, and τ_{xx}, τ_{xz} are estimated according to their largest components. The term under the first integral sign in (2.3), projected on the x-axis, can be estimated as

$$\rho v_n u \sim \rho_\infty U_\infty^2 \delta_*^2 / x^2, \qquad p \cos nx \sim \rho_\infty U_\infty^2 (\delta_* / x)^2 \Delta \tag{2.8}$$

$$T_{nx} \sim \frac{\mu_0 U_\infty}{t}, \qquad \frac{p\cos nx - T_{nx}}{\rho v_n u} \sim \Delta + \frac{\mu_0 x^2}{\rho_\infty U_\infty^2 \delta_*^2 t} \sim \Delta + \left(\frac{\delta_*}{x}\right)^2 \frac{x}{t} \lesssim \Delta + \frac{\delta_*}{x}$$

The characteristic transverse dimension of the t-region, inside which the shock wave interacts with the boundary layer, is certainly not less than δ_* (cf., e.g. [5]), which leads to the last estimate. Thus the second and third term in the integrand can be neglected with respect to the first term. The same argument holds for the second integral.

On the S_3 surface we have $\cos nx \sim \delta_* / x$. On the S_4 surface

$$\cos nx = 0, \qquad \cos ny = 1, \qquad \cos nz = 0 \\ \tau_{nx} = \tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \sim \frac{\mu_0 U_\infty}{\delta_*} \tag{2.9}$$

Taking account of the estimates for the surfaces $S_1 \sim S_2 \sim \delta_* x$, $S_3 \sim S_4 \sim tx$, we have

$$\iint \rho v_n u dS_1 \sim \iint \rho v_n u dS_2 \sim \rho_\infty U_\infty^2 \delta_*^3 / x$$

$$\iint p \cos nx \, dS_3 \sim \rho_\infty U_\infty^2 \delta_*^3 t / x^2, \qquad \iint (p \cos nx - T_{nx}) \, dS_4 \sim \frac{\mu_0 U_\infty}{\delta_*} tx \tag{2.10}$$

From (2.10) it follows that the ratio of the sum of the integrals over S_3 and S_4 to the sum of the integrals over S_1 and S_2 is of order t/x, i.e. tends to zero as R_x tends to infinity. Finally, as $R_x \to \infty$, Equation (2.3), projected on x, takes on the form

$$\iint \rho w u dS_1 - \iint \rho w u dS_2 = 0 \tag{2.11}$$

In an analogous manner Equation (2.4) can be written in the form

$$\iint \rho w i_0 dS_1 - \iint \rho w i_0 dS_2 = 0 \tag{2.12}$$

Equations (2.5), (2.11) and (2.12) are analogues of relations across a discontinuity, into which the t-region contracts in the limit $\delta_*/x \to 0$. The integrals over S_1 can be calculated, as we mentioned before, from the solution for a boundary layer on a flat plate with slip at $\zeta = 0$. Taking account of (1.1), (1.4) these can be reduced to the form

$$J_1 = \iint \rho w \, dS_1 = ({}^4/_5)\, \rho_\infty U_\infty L_*^{3/4} x^{5/4} \sqrt{A} \zeta_0^{1/4} i_1, \qquad L_* = \frac{\mu_0}{\rho_\infty U_\infty}$$

$$J_2 = \iint \rho w u \, dS_1 = ({}^4/_5)\, \rho_\infty U_\infty^2 L_*^{3/4} x^{5/4} \sqrt{A} \zeta_0^{1/4} i_2 \tag{2.13}$$

$$J_3 = \iint \rho w i_0 \, dS_1 = ({}^4/_5)\, \rho_\infty U_\infty^3 L_*^{3/4} x^{5/4} \sqrt{A} \zeta_0^{1/4} i_3$$

$$i_1 = \int_0^\infty W \, d\lambda, \quad i_2 = \int_0^\infty UW \, d\lambda, \quad i_3 = \int_0^\infty gW \, d\lambda$$

The integrals i_1, i_2, i_3 can be calculated from solutions to (1.3). They all converge, due to the exponential vanishing of W as $\lambda \to \infty$.

3. Let us calculate the flow in the Δ-region, using the integral method. Integrate the equations of inviscid flow, in divergence form, over the volume of the Δ_1-region from 0 to x, where x is some fixed value. Take into account that the surface which separates the Δ_1- and Δ_2-regions is a stream surface, and that the mass, momentum, and energy fluxes through sections $z = z_2(x)$ and $z = z_2'(x)$ (Fig.2) can be determined from (2.5), (2.11) and (2.12). The integral relations take on the form

$$\iint \rho u dS_\Delta + 2J_1 = 0, \qquad \iint (p + \rho u^2) \, dS_\Delta - \pi \int_0^x \rho r dr + 2J_2 = 0$$

$$\iint \rho u i_0 dS_\Delta + 2J_3 = 0, \qquad p = \rho_\infty U_\infty^2 k \left(\frac{dr}{dx}\right)^2 \tag{3.1}$$

The factor 2 in front of J_1, J_2, J_3 accounts for the mass, momentum and energy inflow into the Δ_1-region from the both edges of the wing. The double integrals are evaluated over the surface S_Δ, of the Δ_1-region in a given x cross-section. The second integral in the momentum equation is evaluated over the surface l_1 (Fig.2), separating the Δ_1- and Δ_2-regions, $r(x)$ being the radius of the circumference l_1 (Fig.2). In addition to the three conservation equations we have the relation connecting the pressure with the angle of inclination of the surface l_1 with respect to the x-axis, in which k is a constant, assumed to be known (see below). As we showed above, the pressure p in the Δ_1-region is a function of x only. To close the system (3.1) we assume $u = u(x)$, $\rho = \rho(x)$, i.e. we use the simplest version of the integral method. Introduce in (3.1) the dimensionless variables

$$x = L_* x_0,\ r = L_* r_0,\ u = U_\infty u_0,\ p = \rho_\infty U_\infty^2 p_0,\quad \rho = \rho_\infty \rho_0,\quad i_0 = U_\infty^2 i_{00} \tag{3.2}$$

The variable x_0 is clearly identical with R_x.

Taking into account the expressions for J_1, J_2, J_3 (2.13) and the expression $S_\Delta = 0.5\pi r^2$ Equations (3.1) become in dimensionless form

$$\begin{gathered}\rho_0 u_0 r_0^2 = -\nu i_1 x_0^{5/4}, \qquad (p_0 + \rho_0 u_0^2) r_0^2 - 2\int_0^x \rho_0 r_0 r_0' dx_0 = -\nu i_2 x_0^{5/4} \\ \rho_0 u_0 r_0^2 \left(\frac{\varkappa}{\varkappa - 1}\frac{p_0}{\rho_0} + \frac{u_0^2}{2}\right) = -\nu i_3 x_0^{5/4}, \quad p = k(r_0')^2, \quad \nu = \frac{16\sqrt{A_1}\zeta_0^{1/4}}{5\pi}\end{gathered} \tag{3.3}$$

The system of integro-differential equations (3.3) has a simple solution, which satisfies the condition $r_0(0) = 0$. This solution is

$$r_0 = \alpha x_0^{13/16}, \quad p_0 = \beta x_0^{-3/8}, \quad \rho_0 = \gamma x_0^{-3/8}, \quad u_0 = \text{const} \tag{3.4}$$

$$\alpha = \left(\frac{256\nu\,|i_1|\,Z}{169k}\right)^{1/4}, \quad \beta = \frac{13}{16}(k\nu\,|i_1|\,Z)^{1/2}, \quad \gamma = \frac{13}{16\,(i_2/i_1 + 0.3Z)}\left(\frac{\nu k\,|i_1|}{Z}\right)^{1/2} \tag{3.5}$$

$$u = \frac{i_2}{i_1} + 0.3Z,\ Z = \frac{\sqrt{\varkappa^2 i_2^2 + [0.18(\varkappa-1)^2 + 1.2\varkappa(\varkappa-1)]\,i_1 i_3} - [\varkappa + 0.3(\varkappa-1)]\,i_2}{[0.09(\varkappa-1) + 0.6\varkappa]\,i_1}$$

The value of the constant k is found from numerical calculations of the self-similar solution for the flow over an axisymmetric power-law body of the form $y \sim x^n$, $n = 13/16$.

Following is a table of values of α, β, γ, u_0 and T (T is the temperature in the Δ_1-region, scaled with respect to the stagnation temperature) for $\varkappa = 1.4$ and $\varkappa = 1.667$ and leading edge sweep angle $\omega = 60°$. The values of i_1, i_2 are based on calculations performed by A.A.Bogacheva, some results of which were cited in [1]. The calculations were performed for an insulated surface, for which, as can be easily seen, $i_3 = 0.5 i_1$.

ω	α	β	γ	u_0	T
Diatomic gas ($\varkappa = 1.4$, $k = 0.950$)					
30°	0.548	0.189	2.55	0.694	0.518
45°	0.612	0.235	3.18	0.694	0.518
60°	0.664	0.277	3.69	0.689	0.525
75°	0.678	0.288	3.80	0.685	0.530
Monatomic gas ($\varkappa = 5/3$, $k = 0.982$)					
30°	0.590	0.226	2.35	0.721	0.479
45°	0.661	0.283	2.91	0.716	0.487
60°	0.713	0.330	3.34	0.711	0.494
65°	0.723	0.339	3.56	0.703	0.505

It can be seen from the examples that, in fact, $u \sim U_\infty$ and $T \sim 1$ in the Δ_1-region, which confirms the estimates (1.6).

Fig.3 shows the pressure p_0 and the thickness of the effective body as functions of $x_0 = R_x$ for $\omega = 60°$, calculated from (3.4), (3.5). The solid and dashed lines correspond to $\varkappa = 1.4$, $\varkappa = 1.667$, respectively. For comparison, there are also given the dimensionless pressure p_1 and the thickness of the boundary layer y_1, calculated from

$$p_1 = \frac{A}{\sqrt{R_x \cot \omega}}$$

$$y_1 = B\,(R_x \cot \omega)^{3/4}$$

which follow from the solution for a slipping plate at $z = 0$.

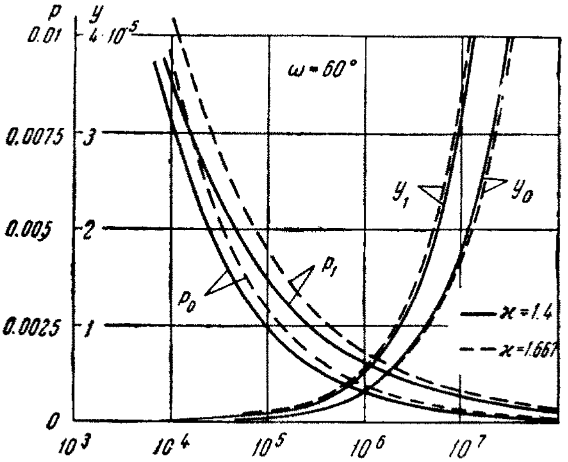

Fig. 3

From Fig.3 one can see how the pressure and the boundary-layer thickness near the plane of symmetry of the wing increase as compared with the values computed from the solution for a plate with slip.

The solution obtained above is of asymptotic nature. Rigorously speaking it may be used for $p_1 \gg p_0$, and $y_1 \gg y_0$. One can assume that the basic qualitative features are valid already for $y_1 > y_0$ and $p_1 > p_0$.

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M.D.LADYZHENSKII

(November 10, 1931 — May 15, 1965)

Mikhail Davidovich Ladyzhenskii was born to a soldier's family in Odessa. In 1949 he began his studies at Moscow University in the Physico-Technical Department, which two years later became an autonomous Institute. After graduating with honors from the Institute he joined in 1955 the Central Aero-Hydrodynamic Institute (TsAGI).

In 1961 he submitted his thesis for the degree of Candidate of Physico-Mathematical Sciences, entitled "Some Problems in the Gas Dynamics of

Hypersonic Flows".

M.D.Ladyzhenskii's interests covered a very wide field. He has worked on the theory of propagation of shock waves from supersonic planes, and on the theory of three-dimensional hypersonic flow, in particular on three-dimensional flow past slender weakly-blunted bodies (hypersonic area rule). Several of his investigations were devoted to problems of control of flow of an ionized gas in a magnetic field at low magnetic Reynolds numbers.

Problems of high-altitude hypersonic flight are connected with the study of viscous hypersonic flows. M.D.Ladyzhenskii was a pioneer in the theory of three-dimensional hypersonic flow of a viscous gas, and this theory has been published only as far as it has been advanced by his work.

In 1961 he was awarded the N.E.Zhukovskii Prize of the First Class (jointly with O.M.Belotserkovskii and V.V.Sychev) for his investigations in the field of hypersonic nozzles and three-dimensional hypersonic flows.

In addition to his work in the N.E.Zhukovskii Central Aero-Hydrodynamic Institute, M.D.Ladyzhenskii was on the faculty of the Moscow Physico-Technical Institute.

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